

Delay Embedding on Time Series

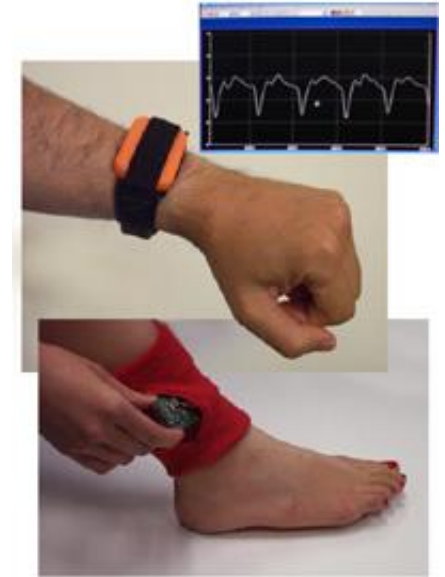
Zhifei Zhang

Outline

- 1. Why delay embedding?**
- 2. How delay embedding works?**
- 3. Modeling and classification**
- 4. Experimental evaluation**

Why delay embedding?

- ❖ Time-varying signal (**Online**)
- ❖ Real-time processing (**Real-time**)
- ❖ Limited storage (**Small memory**)

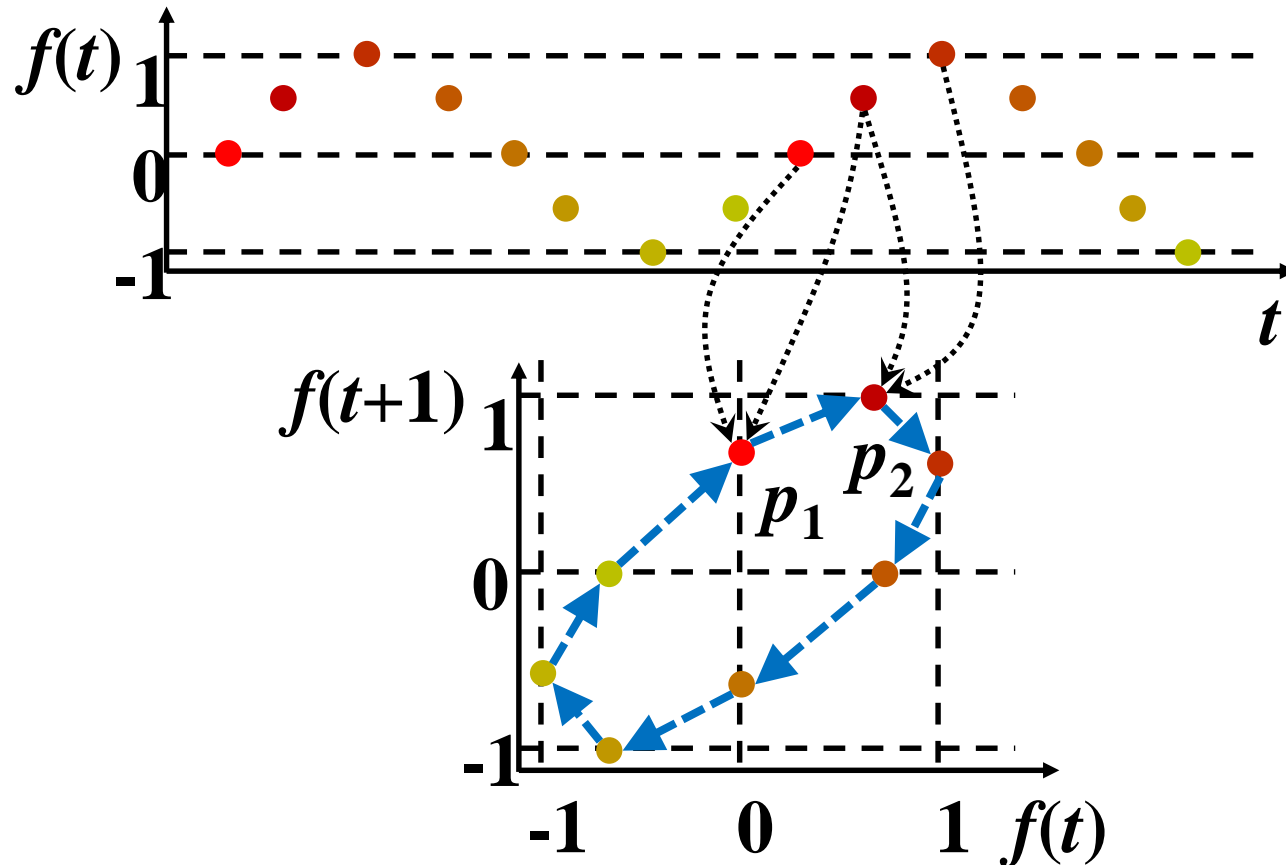


Why delay embedding?

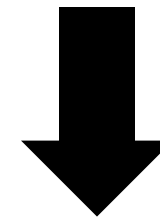
**Online, real-time
and small memory
fails most existing
works.**

Algorithm	Fixed length	Well aligned	Complete pattern	Off-line
DTW	×		×	×
1NN-DTW	×		×	×
kNN	×	×	×	×
HMMs	×	×		×
Decision tree	×		×	×
SVM	×		×	×
Neural network	×		×	×
Dictionary			×	×
Sparse coding				×

Why delay embedding?

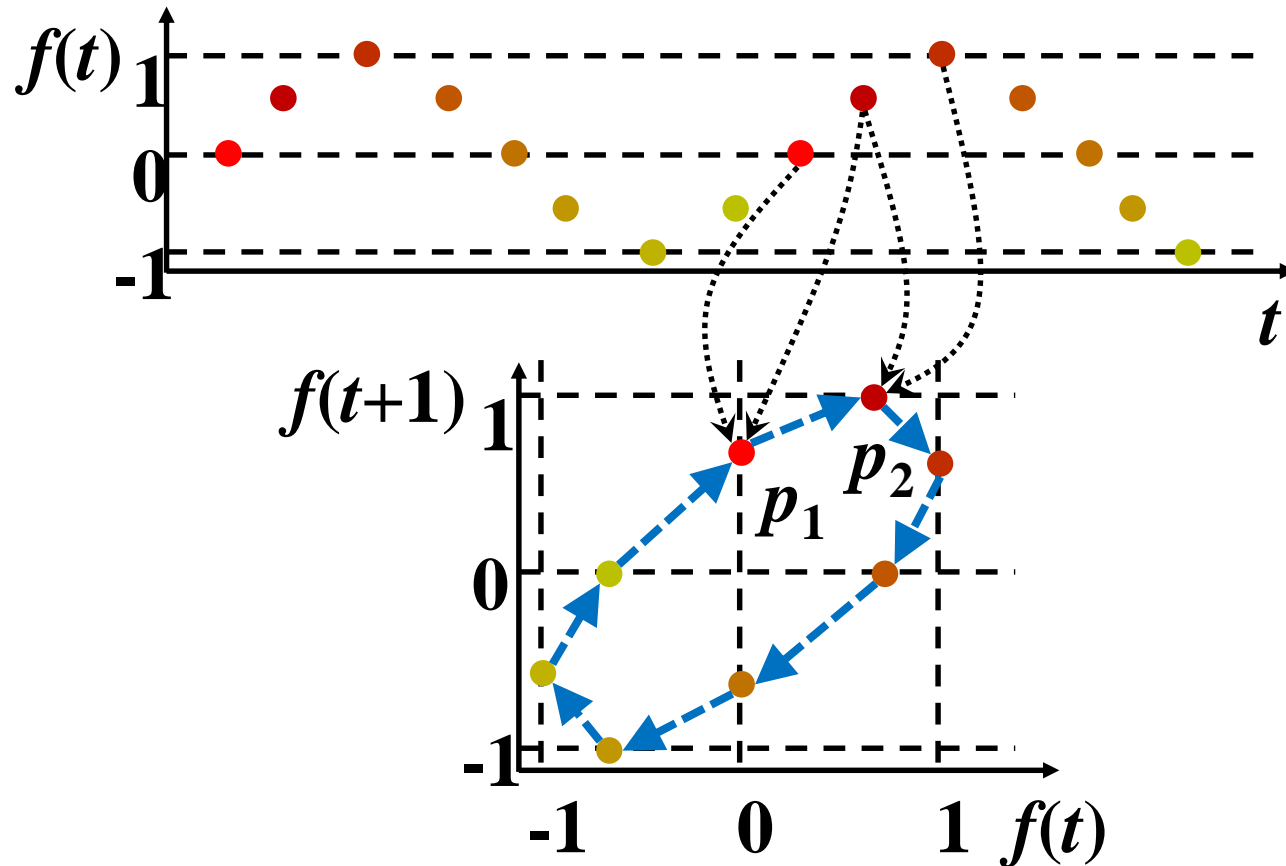


**1-D time series
(time space)**



**2-D time series
(embedding space)**

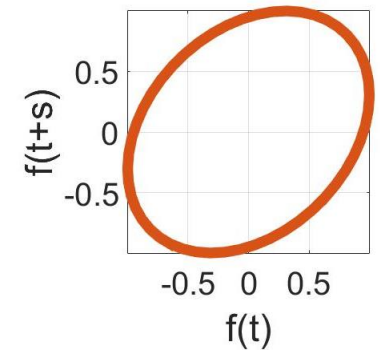
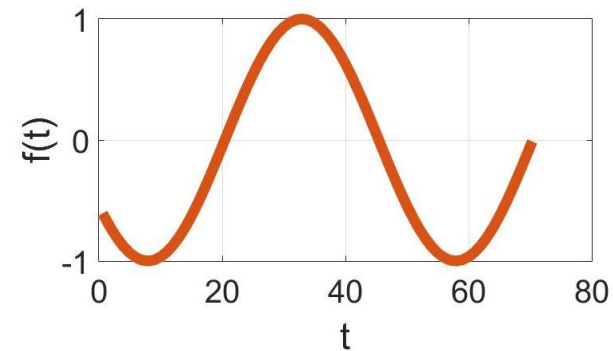
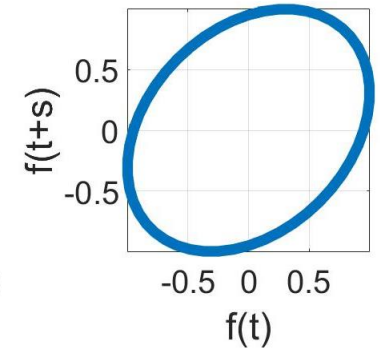
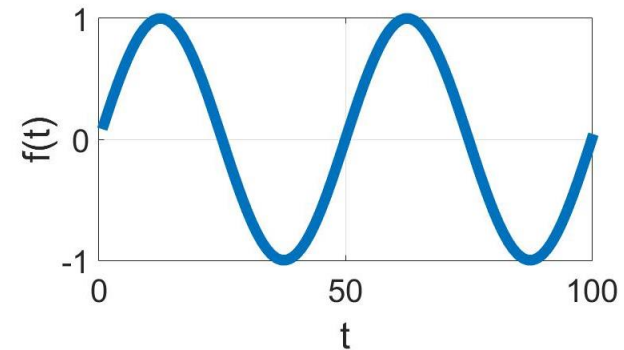
Why delay embedding?



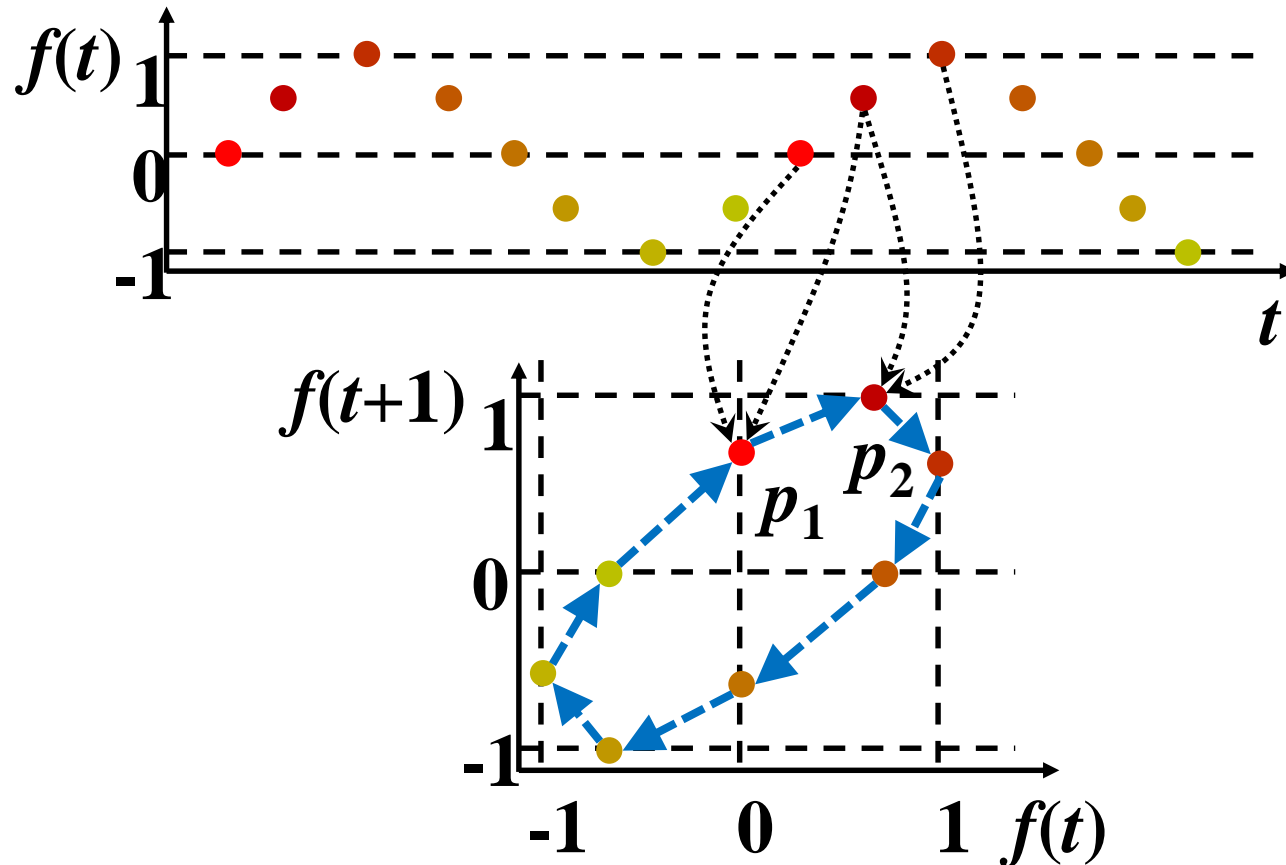
Online

Real-time

Small memory



Why delay embedding?



Online

Real-time

Small memory

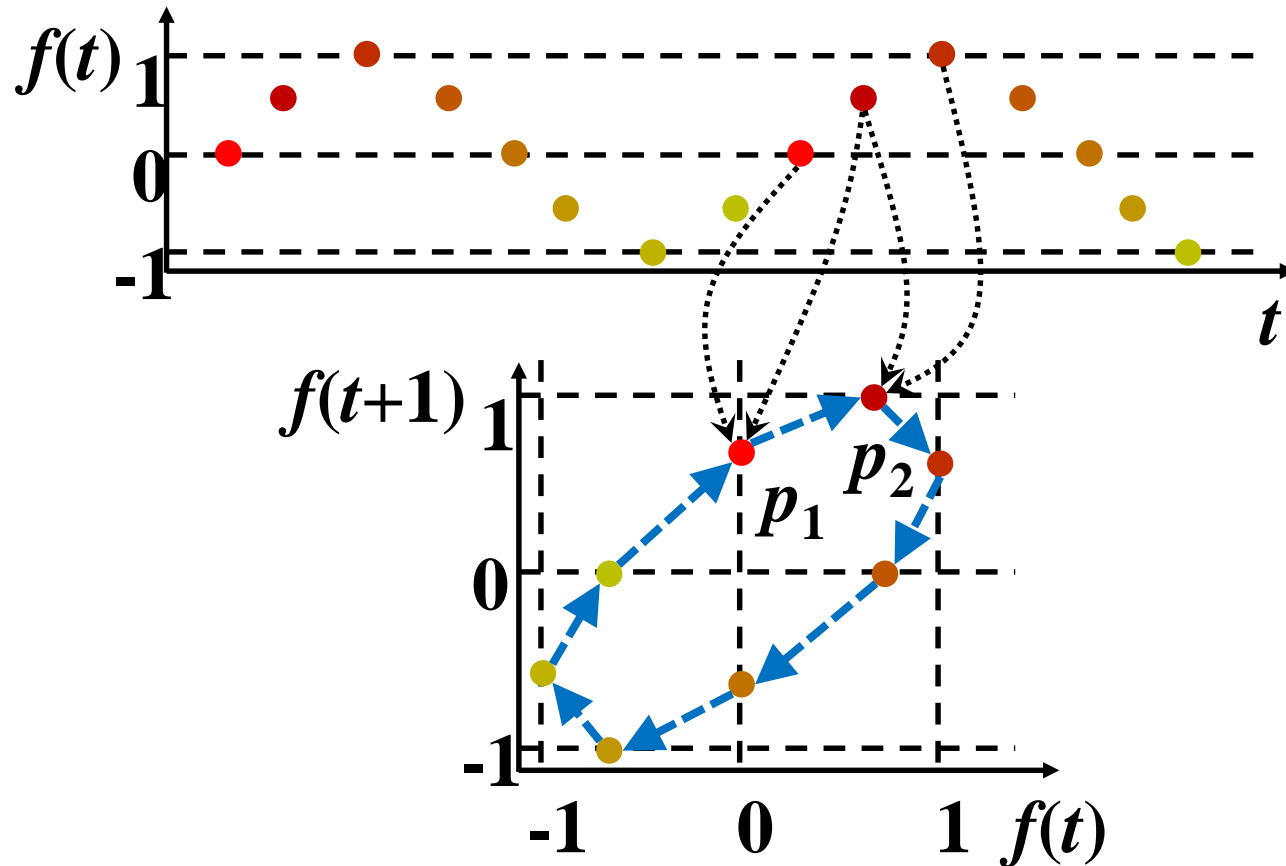
$[t, f(t)]$

$[t+1, f(t+1)]$

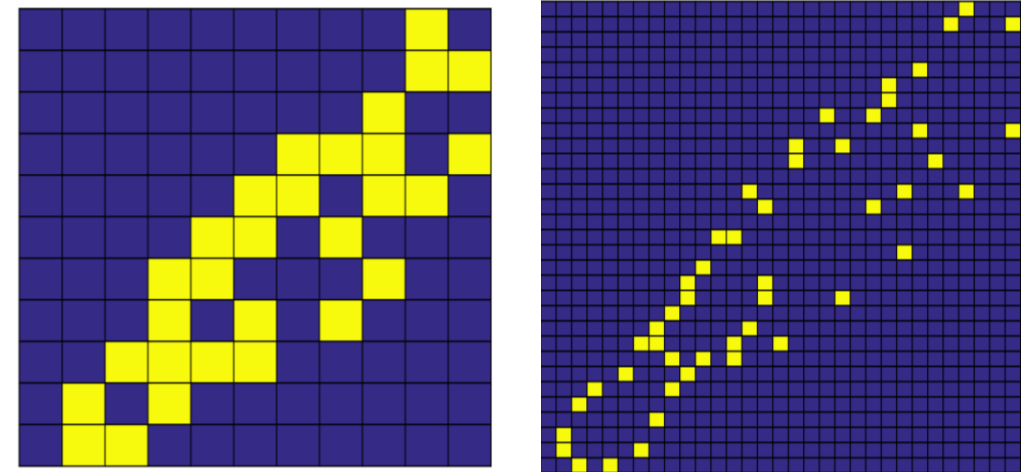
↓ $O(1)$

$[f(t), f(t+1)]$

Why delay embedding?



Online
Real-time
Small memory



Discretization

How delay embedding works?

For different patterns,
the trajectories are
distinguishable

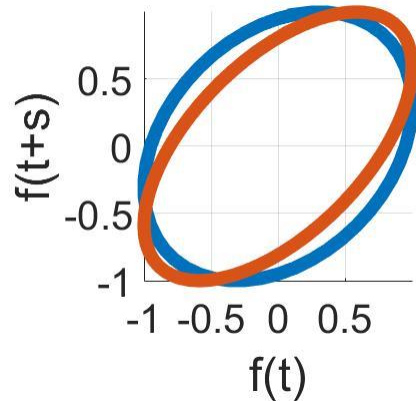
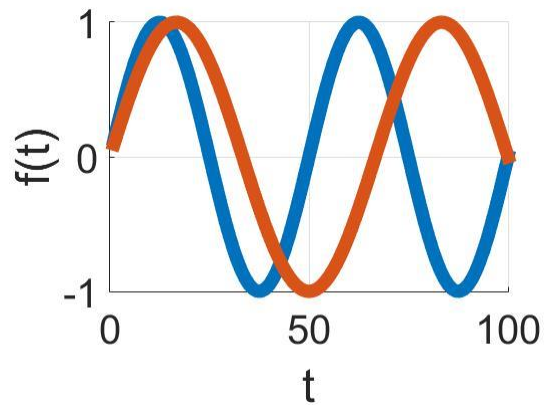
- **Frequency**
- **Amplitude**
- **Tendency**

For the same pattern,
the trajectories are
invariant

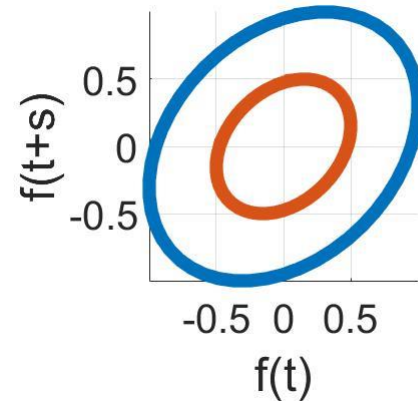
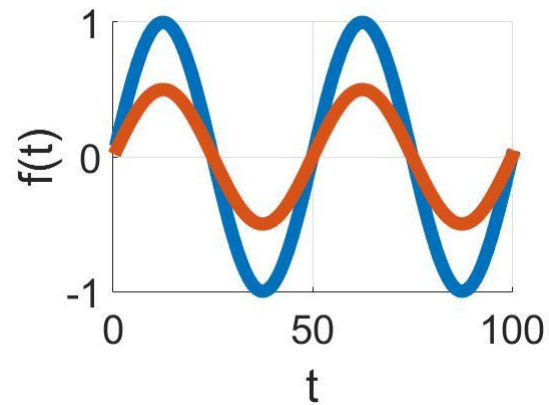
- **Length**
- **Phase**
- **Baseline**

How delay embedding works?

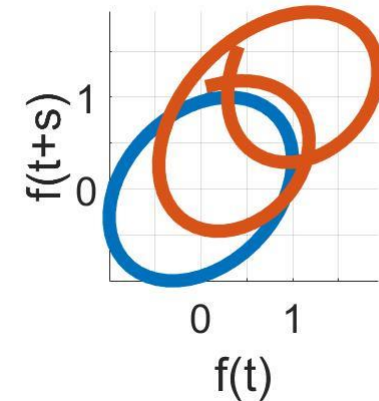
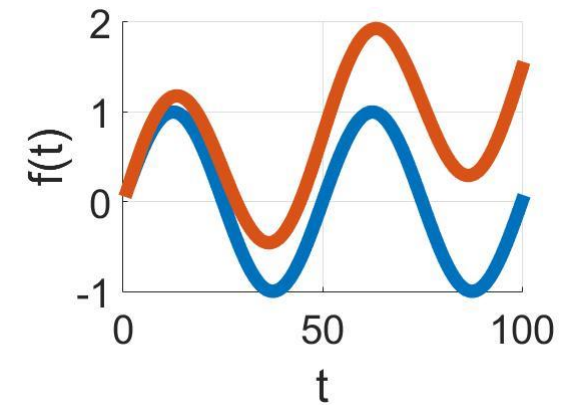
Frequency



Amplitude

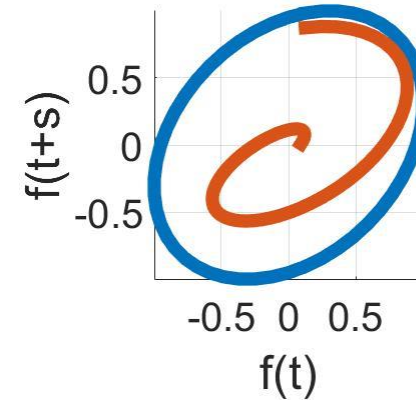
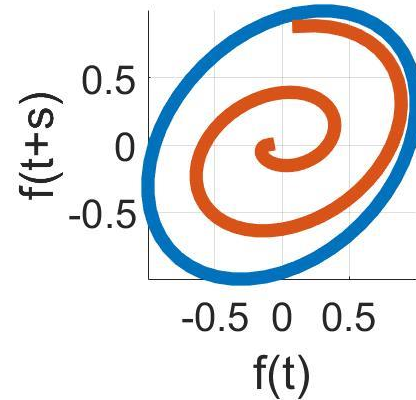
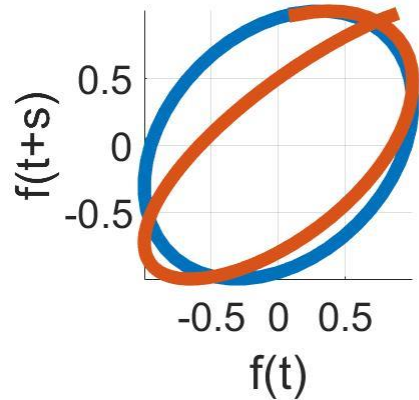
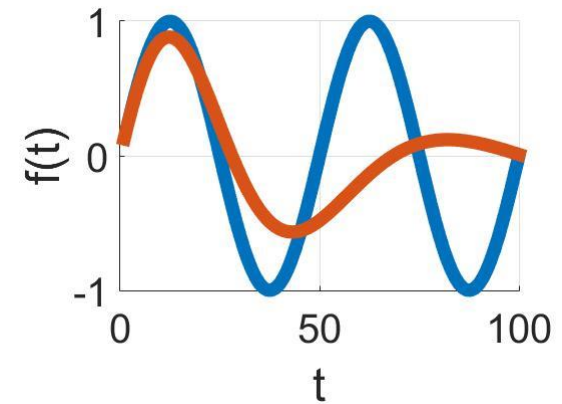
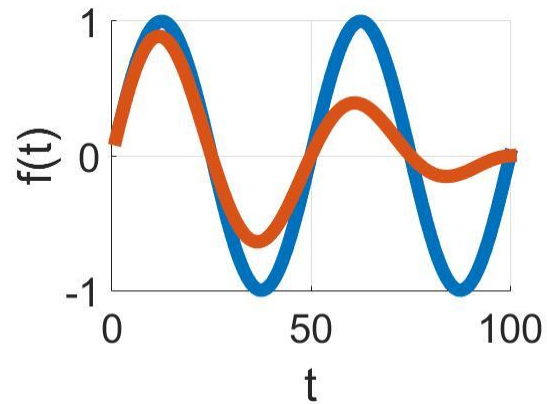
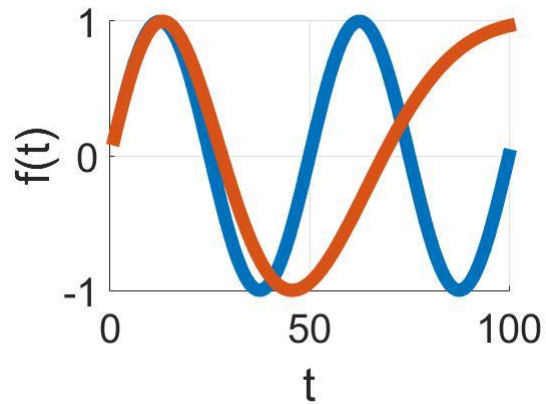


Tendency



How delay embedding works?

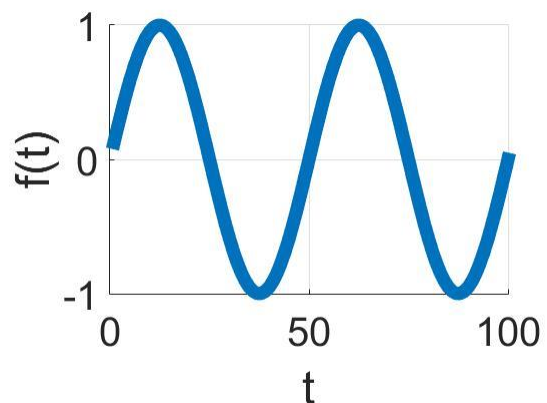
Attenuation in frequency, amplitude and both



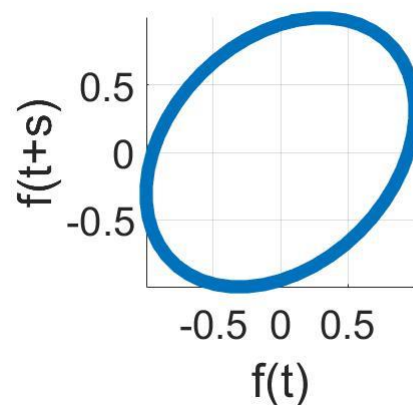
How delay embedding works?

Some other ways to perform delay embedding

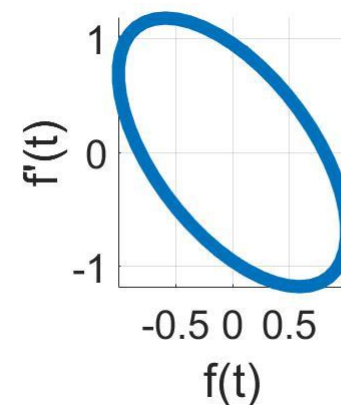
Signal



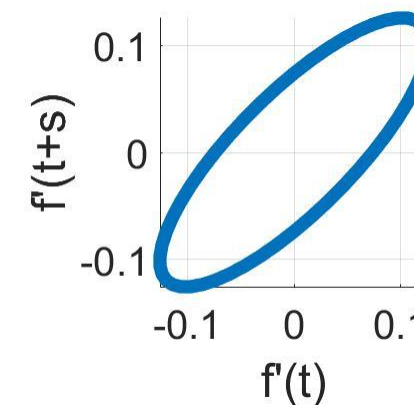
Delay Embedding



Differential Embedding



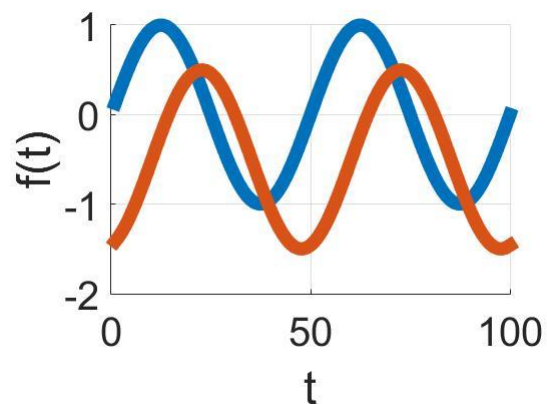
Derivative Delay Embedding



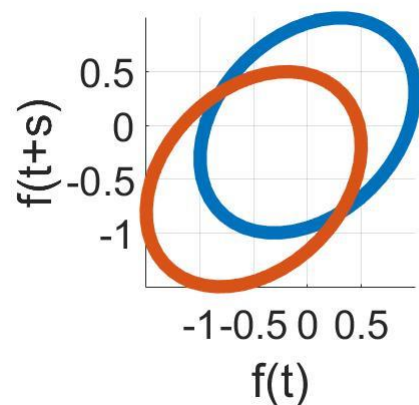
How delay embedding works?

Some other ways to perform delay embedding

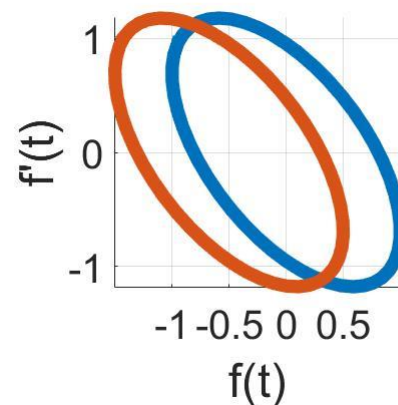
Signal



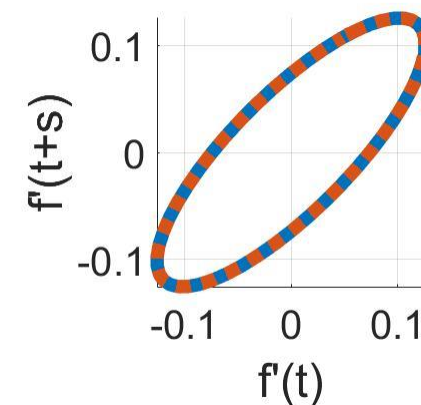
Delay Embedding



Differential Embedding

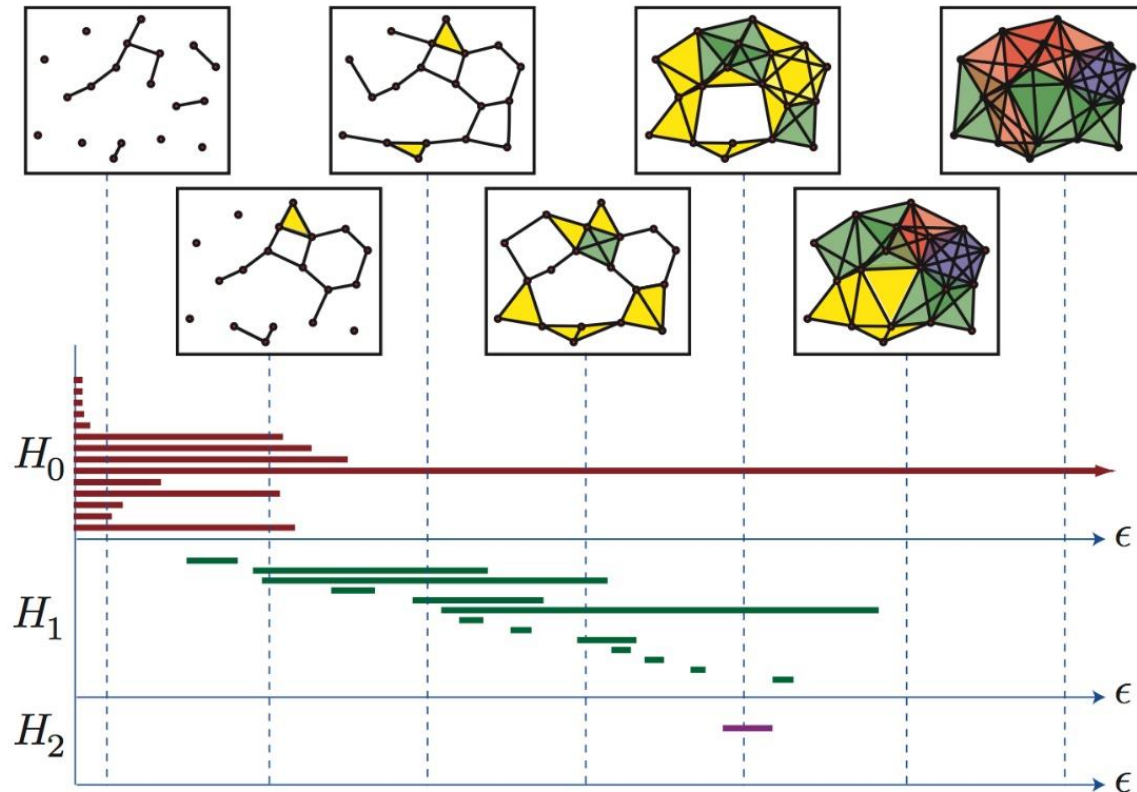


Derivative Delay Embedding

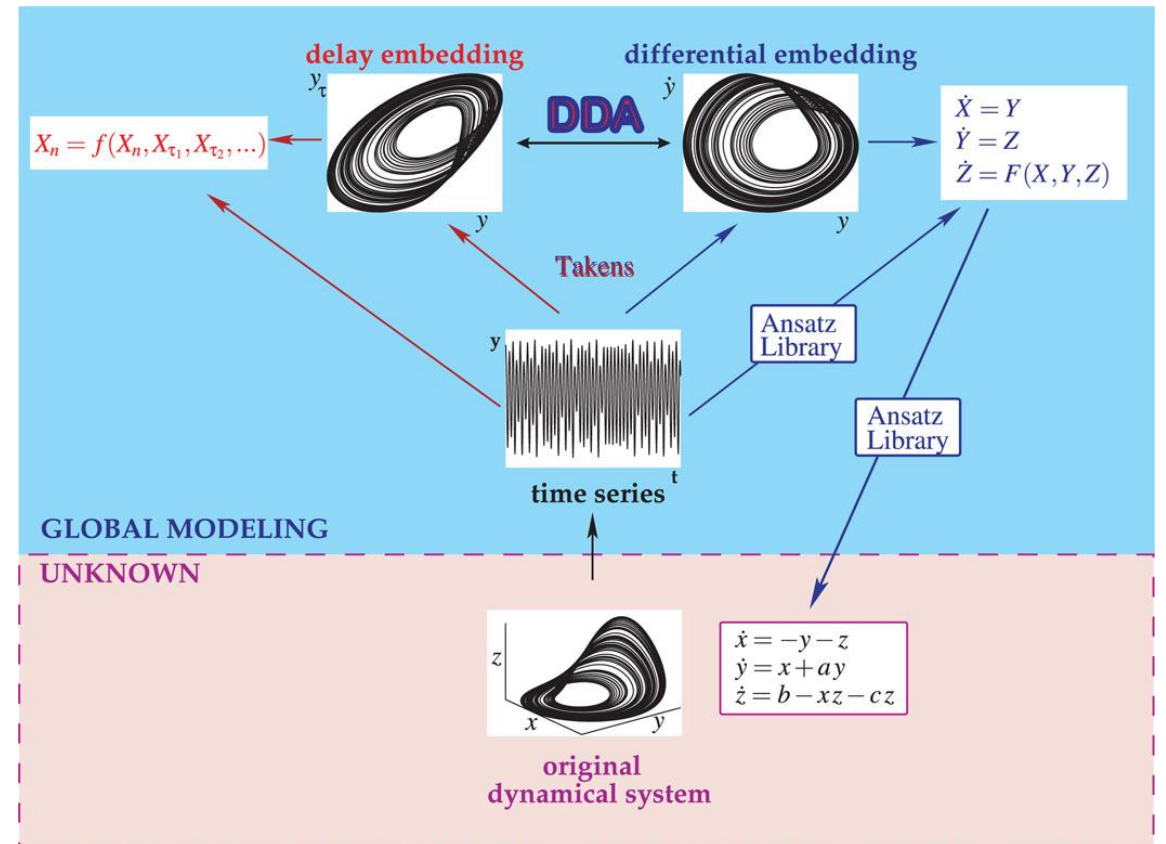


Modeling and Classification

Persistent Homology



Differential Equations



Modeling and Classification

Persistent Homology

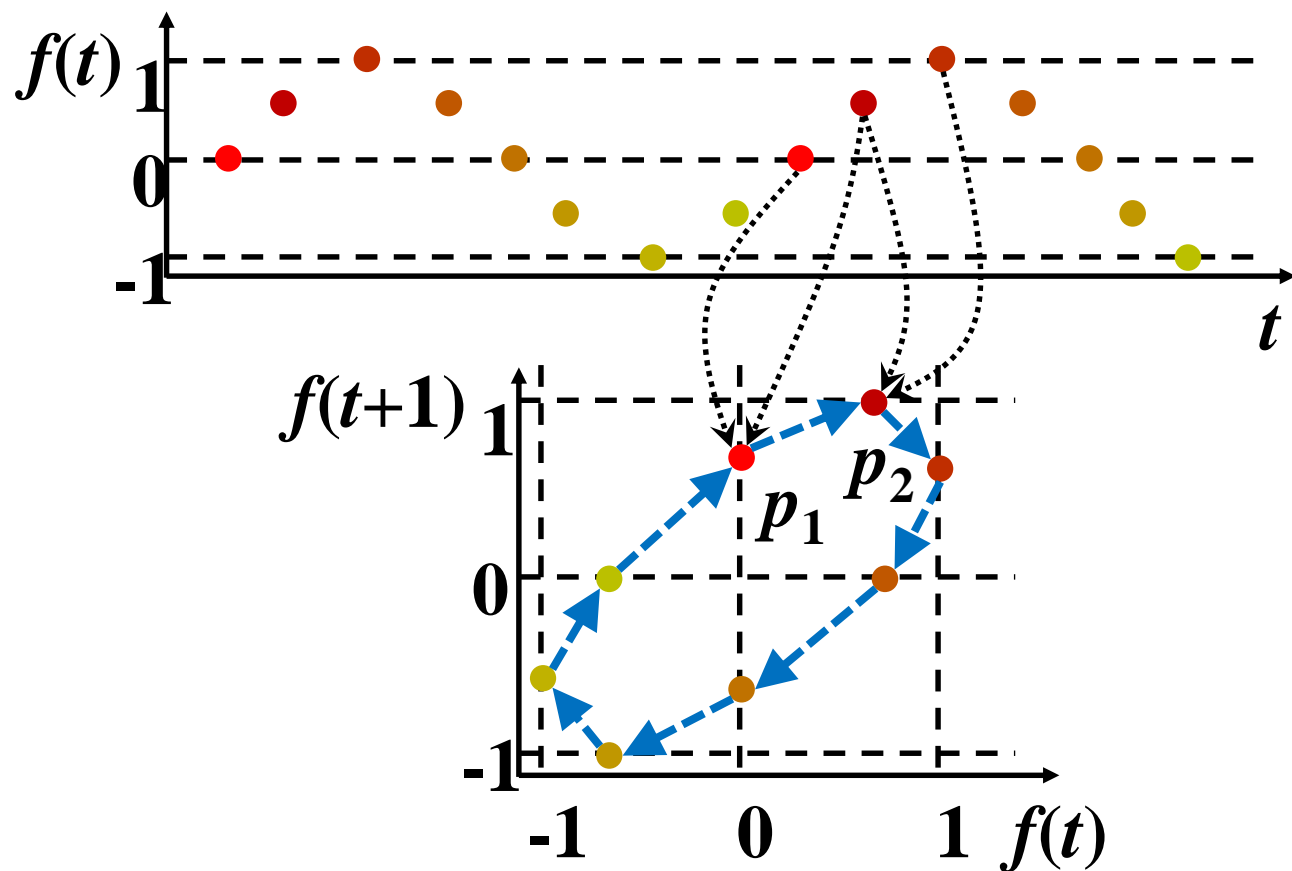
- **Periodical patterns**
- **Time-consuming**
- **Low fidelity**

Differential Equations

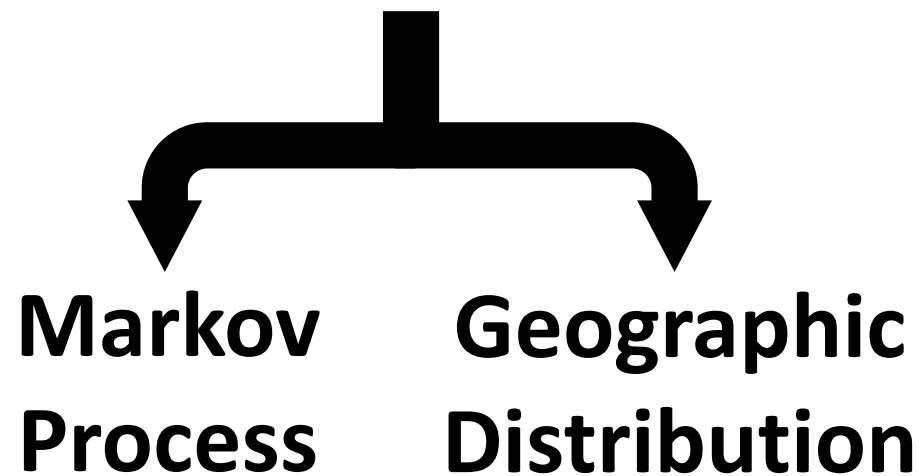
- **Smooth trajectory**
- **Predefined equations**
- **Off-line**

**To achieve online processing, we may simply
record and match trajectories**

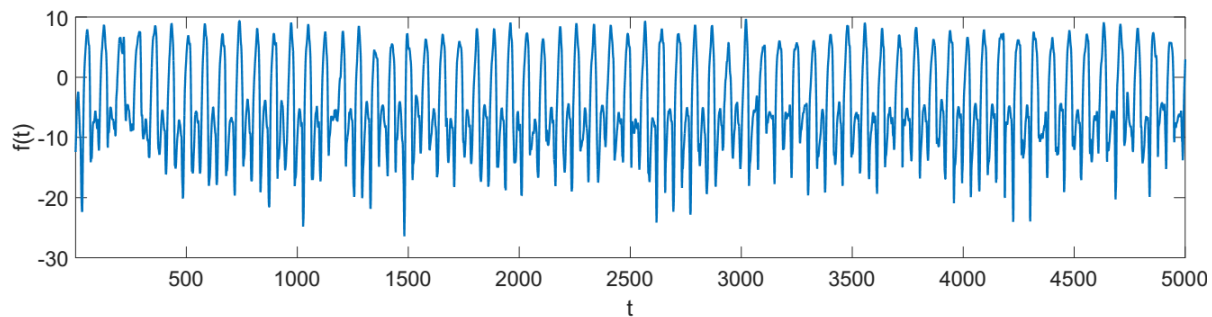
Modeling and Classification



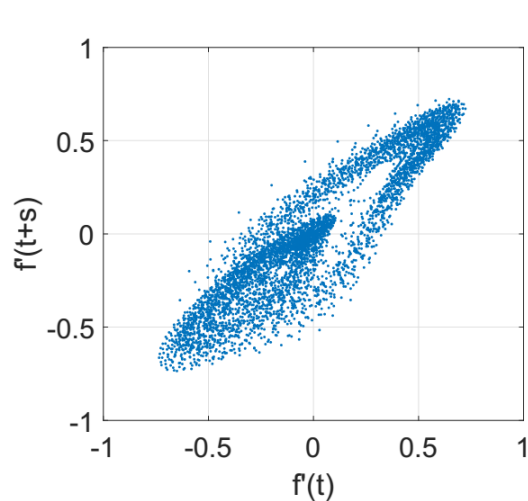
Directed Graph



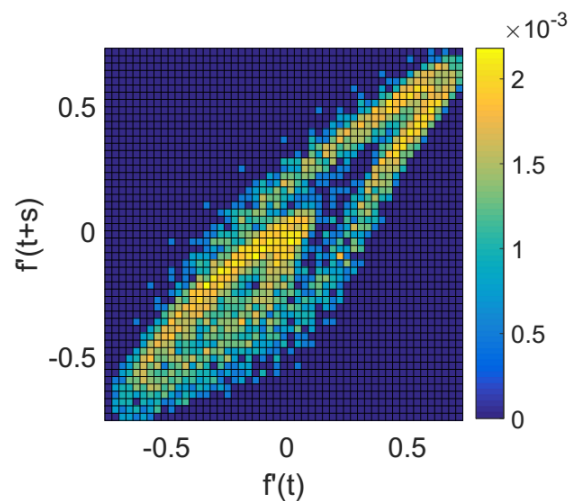
Modeling and Classification



(a) Time series

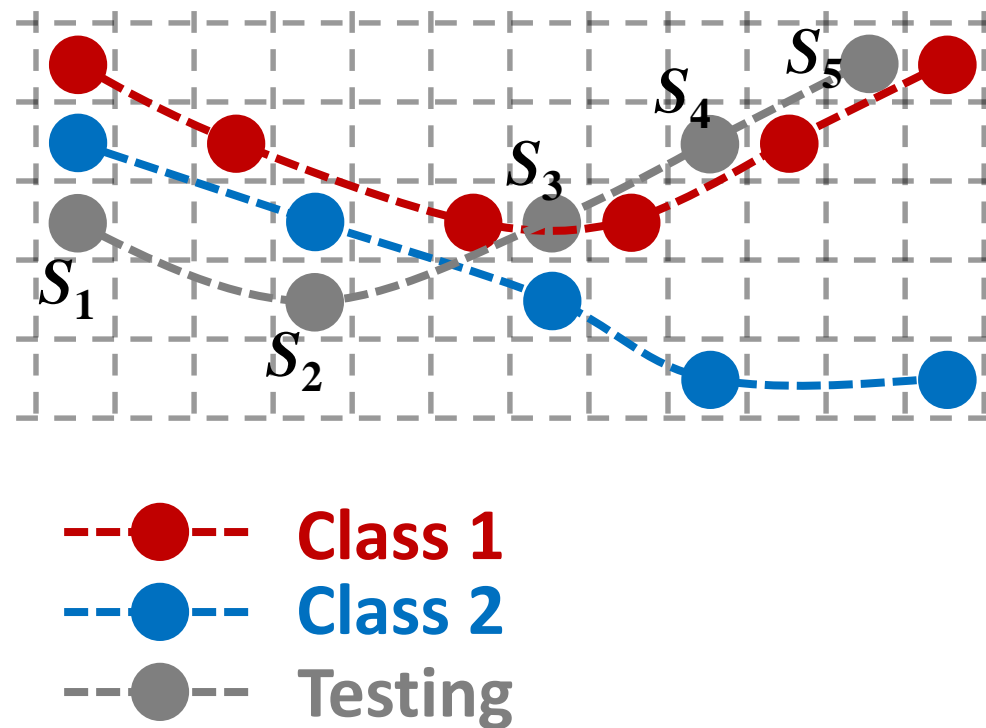


(b) Point cloud



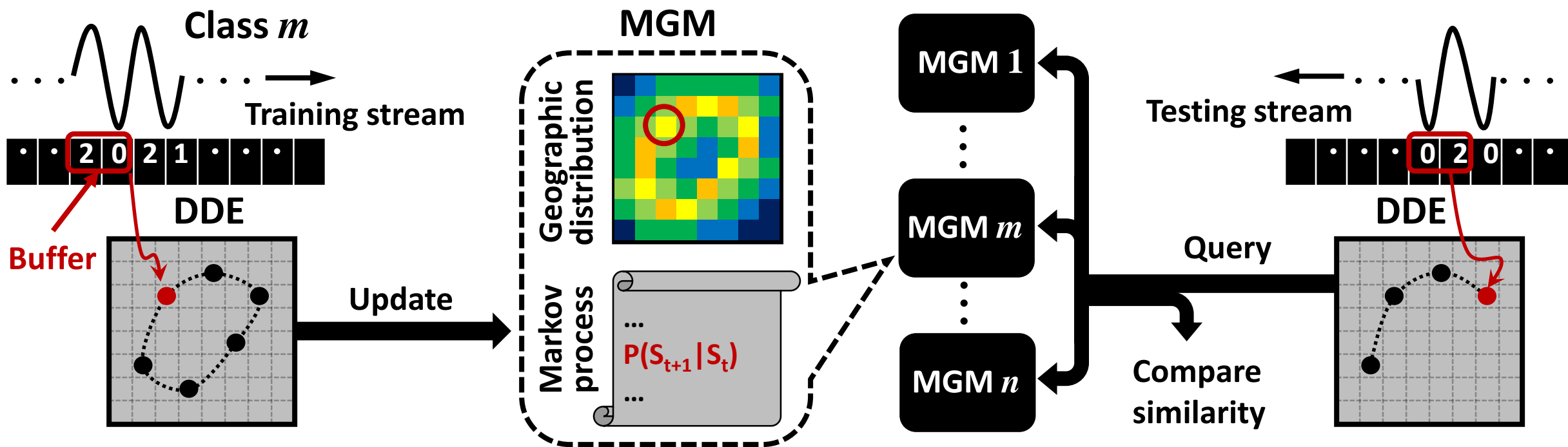
(c) Geographic distribution

Toy example of classification

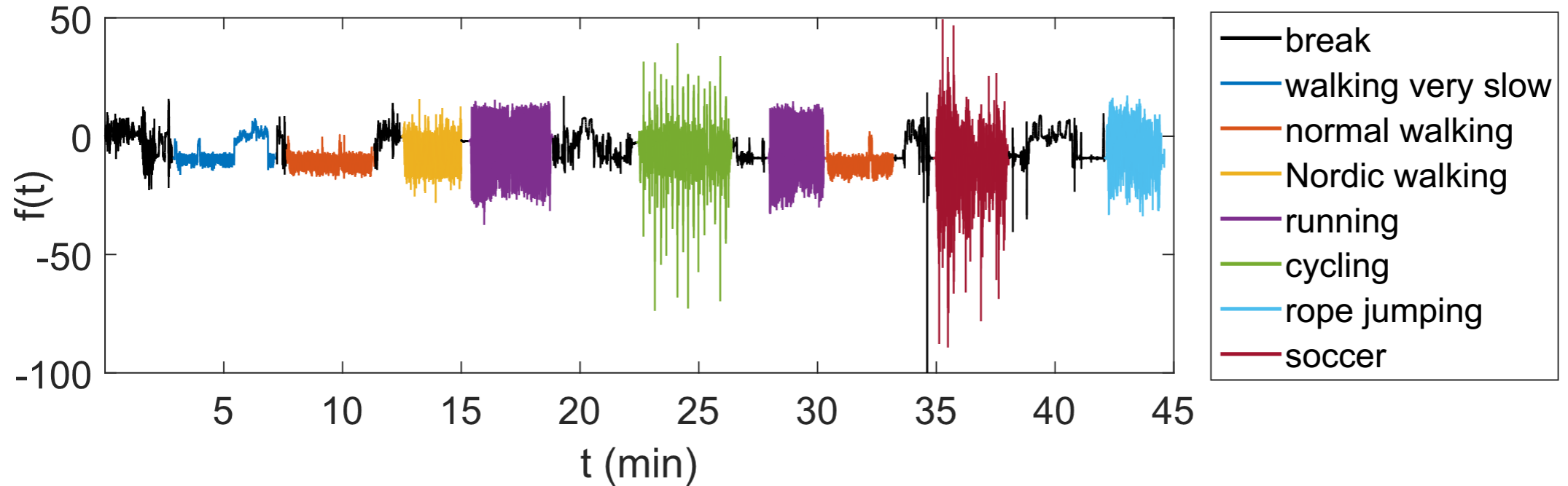


Modeling and Classification

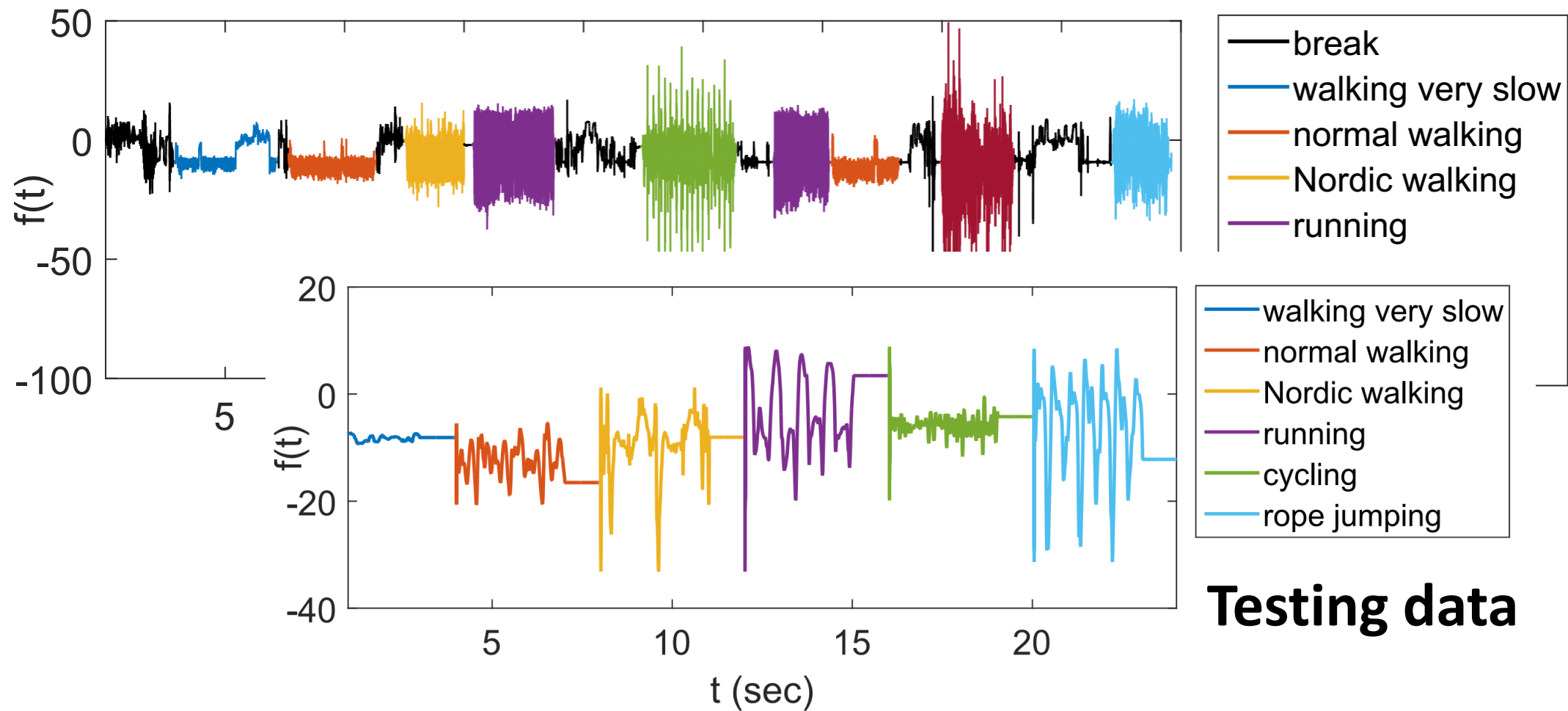
DDE-MGM Scheme



Experimental Evaluation



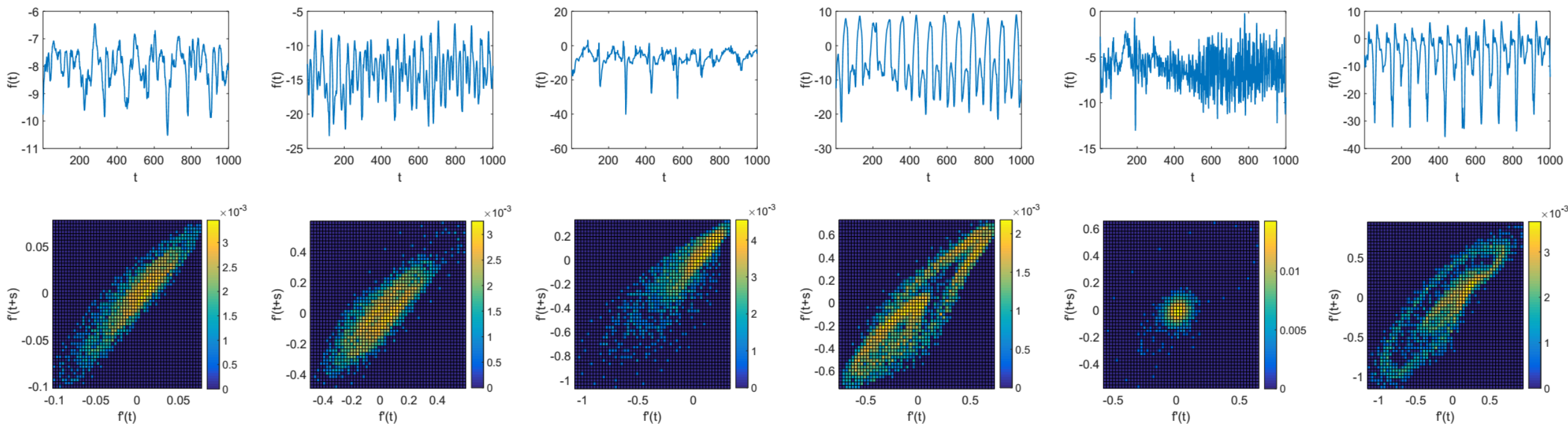
Experimental Evaluation



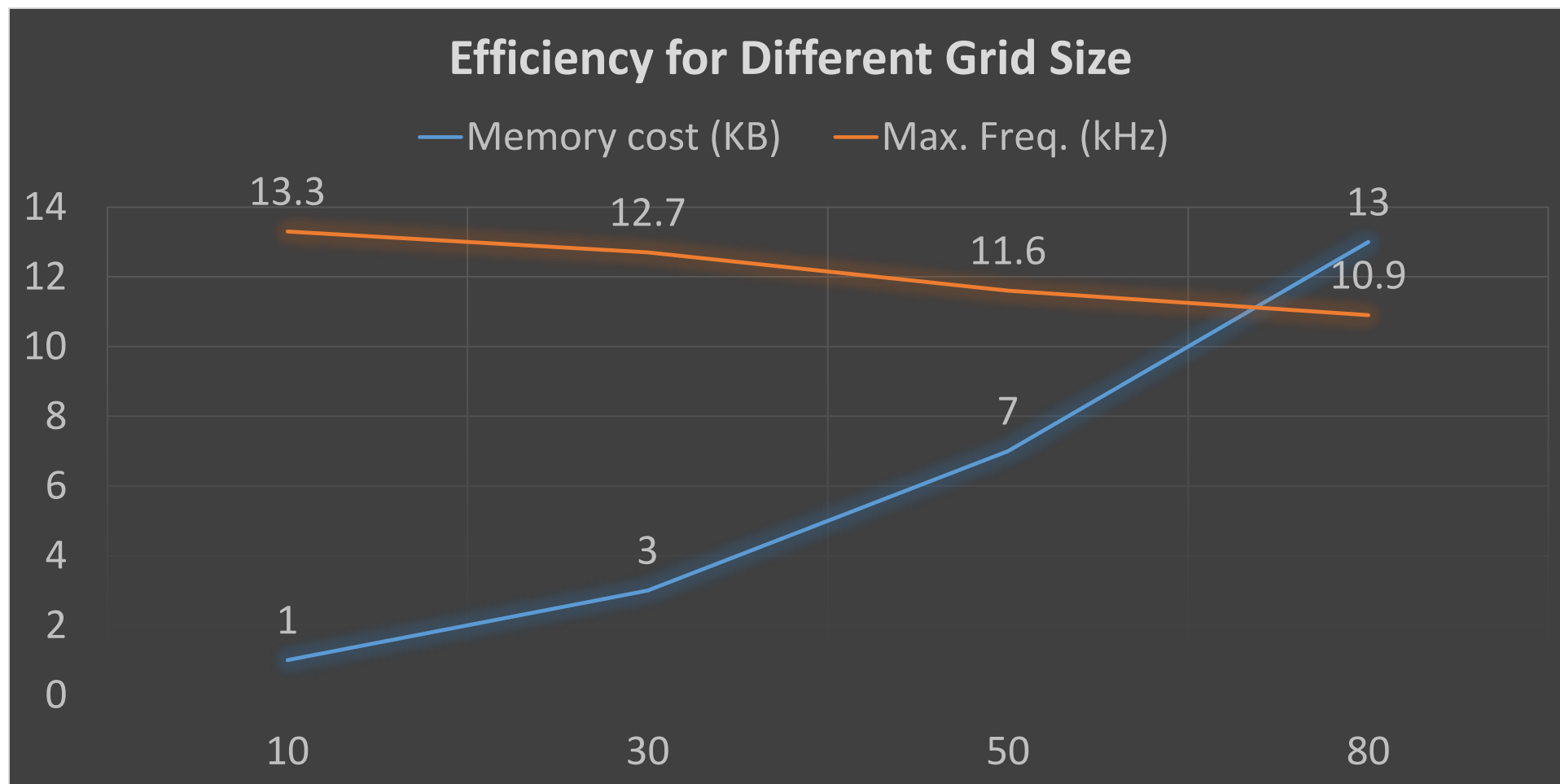
Testing data

Experimental Evaluation

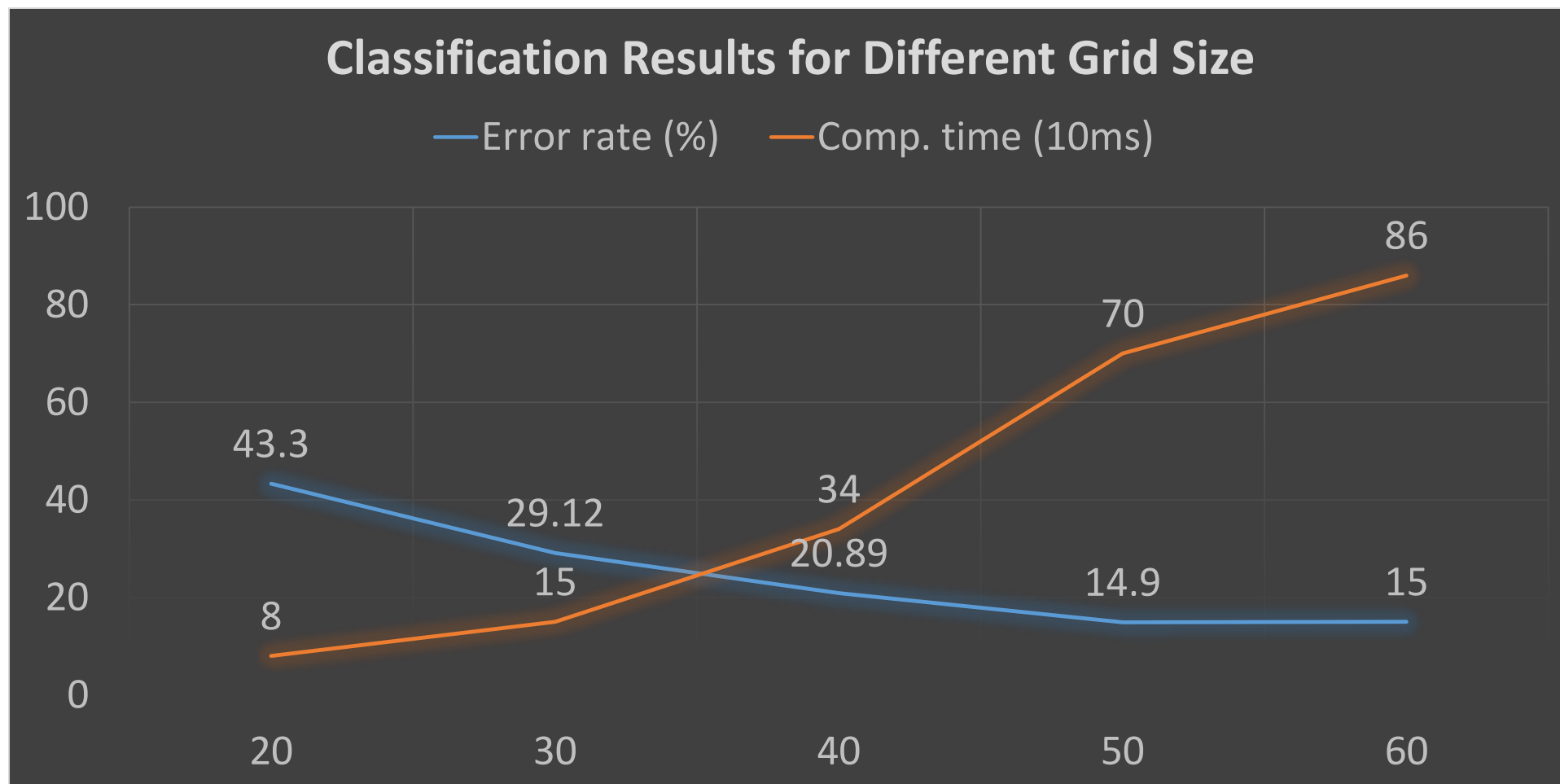
Example segments and geographic distribution



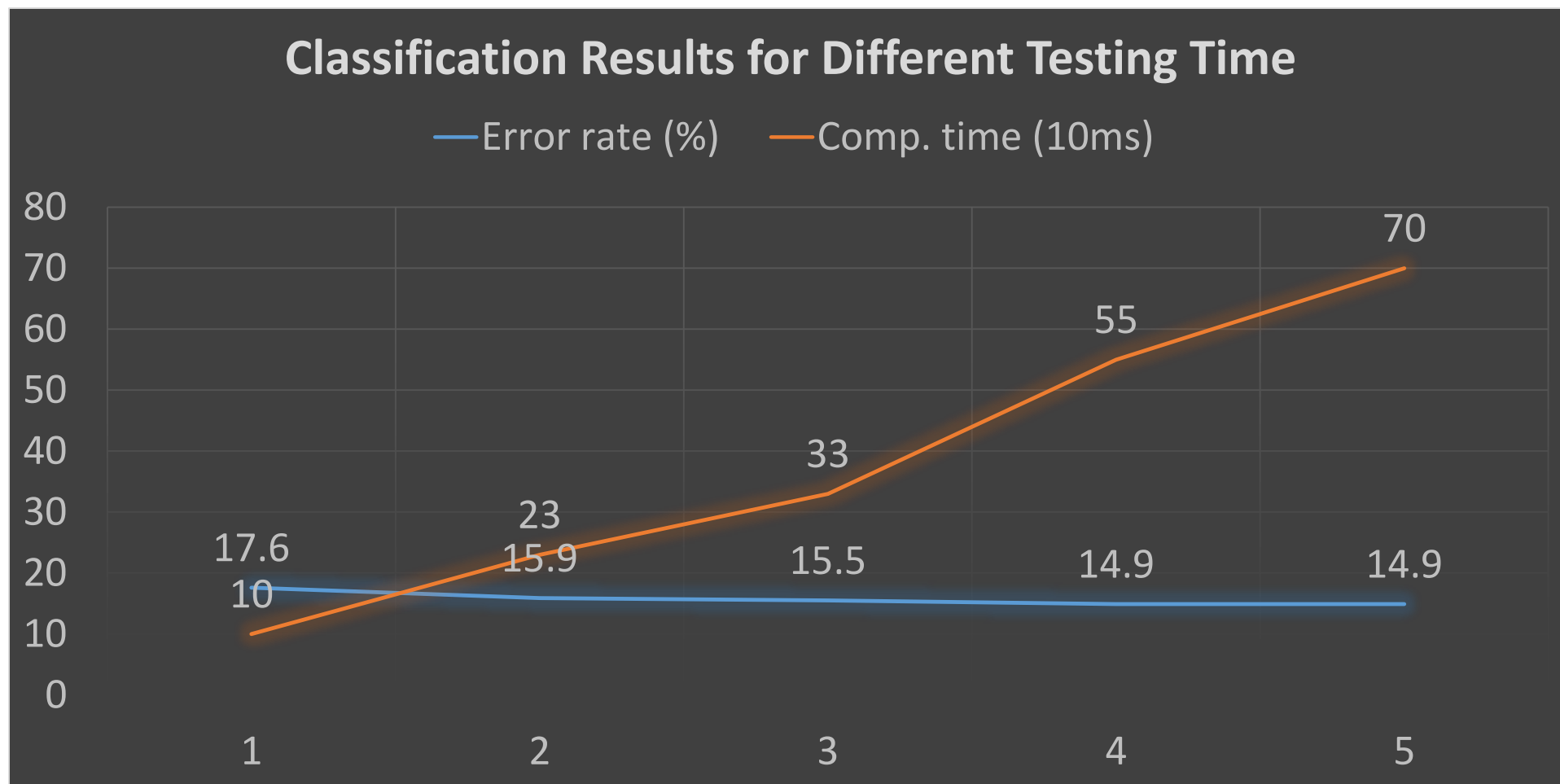
Experimental Evaluation



Experimental Evaluation

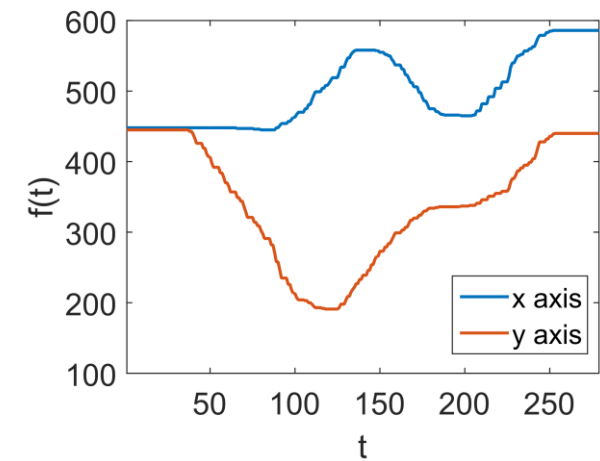
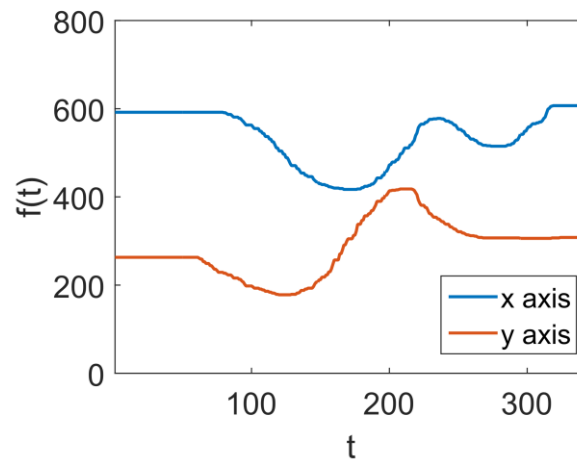
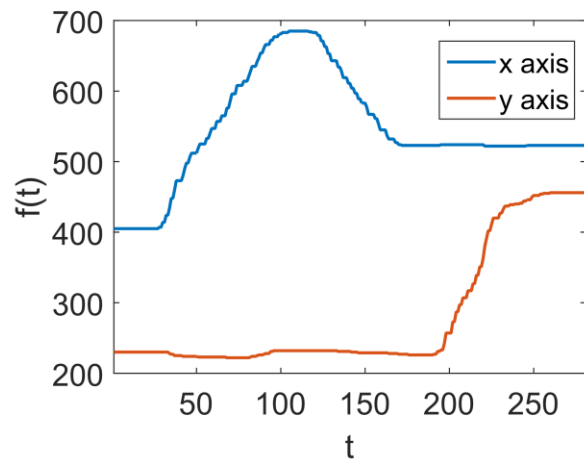


Experimental Evaluation



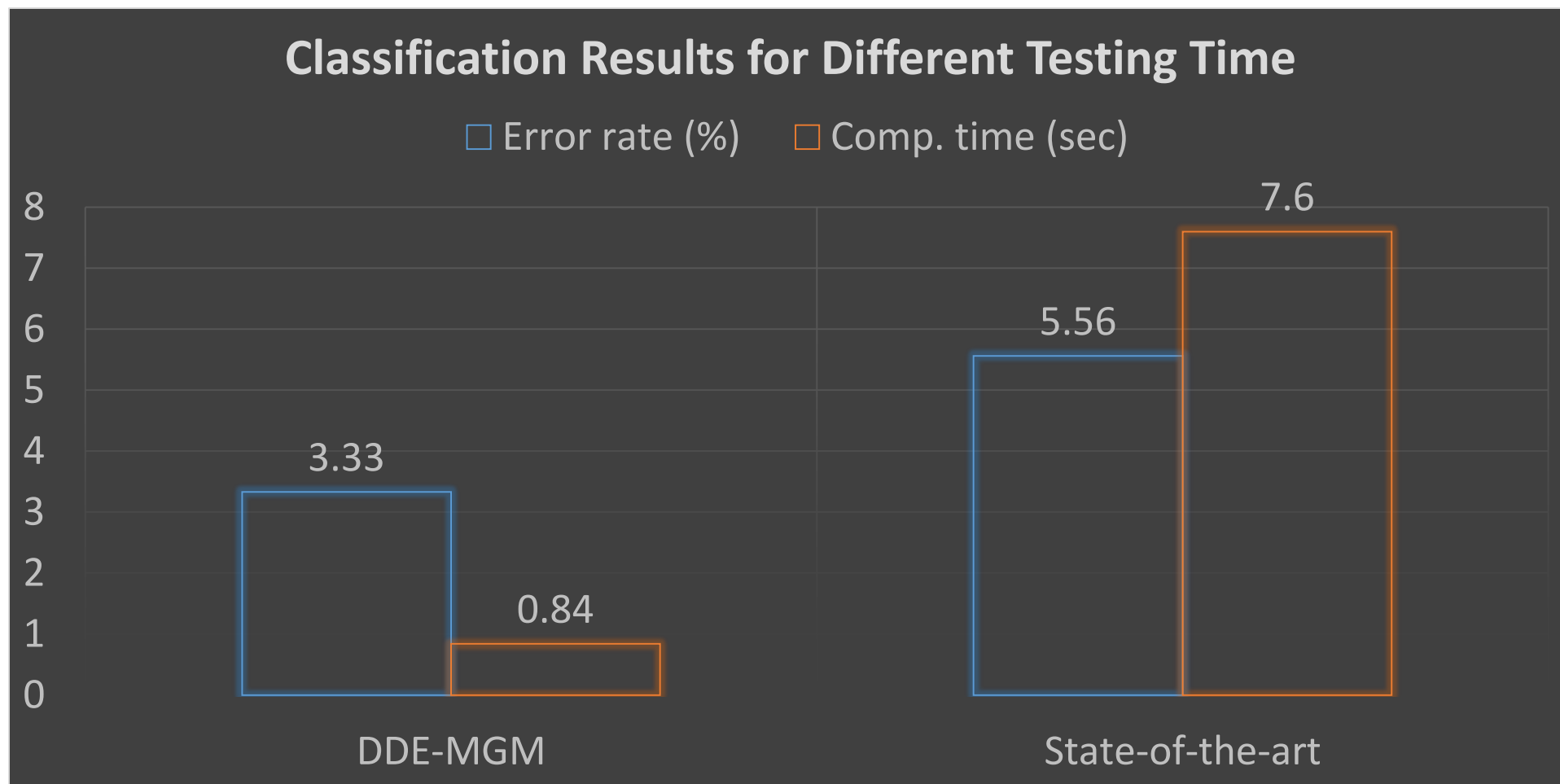
Experimental Evaluation

DDE-MGM VS. DTW-base Approach



Leave-20%-out cross validation \times 100 iterations

Experimental Evaluation



Thank You